SIR CHHOTU RAM INSTITUTE OF ENGINEERING AND TECHNOLOGY

DEPARTMENT OF MECHANICAL ENGINEERING

ENGINEERING AND MECHATRONICS (KME-101T)

Unit-1: Introduction to Mechanics of Solid

Stress (σ)

Every material is elastic in nature. That is why, whenever some external system of forces acts on a body, it undergoes some deformation. As the body undergoes deformation, its molecules set up some resistance to deformation. This resistance per unit area to deformation, is known as stress. Mathematically stress may be defined as the force per unit area *i.e.*, stress.

$$\sigma = \frac{P}{A}$$

where P = Load or force acting on the body, and

A = Cross-sectional area of the body.

In S.I. system, the unit of stress is pascal (Pa) which is equal to 1 N/m^2. In actual practice, we use bigger units of stress *i.e.*, megapascal (MPa) and gigapascal (GPa), which is equal to N/mm^2 or kN/mm^2 respectively.

Strain (ε)

As already mentioned, whenever a single force (or a system of forces) acts on a body, it undergoes some deformation. This deformation per unit length is known as strain. Mathematically strain may be defined as the deformation per unit length. *i.e.*, strain

$$\varepsilon = \frac{\delta l}{l} \quad \text{or} \quad \delta l = \varepsilon\, l$$

where δl = Change of length of the body, and

l = Original length of the body.

Types of Stresses

Though there are many types of stresses, but we shall study the following two types of stresses in this chapter:

1. Tensile stress. **2.** Compressive stress.

Tensile Stress

When a section is subjected to two equal and opposite pulls and the body tends to increase its length, as shown in Fig. 28.1, the stress induced is called tensile stress. The corresponding strain is called tensile strain. As a result of the tensile stress, the *cross-sectional area of the body gets reduced.

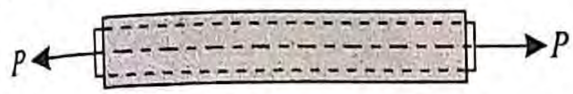


Fig. Tensile stress

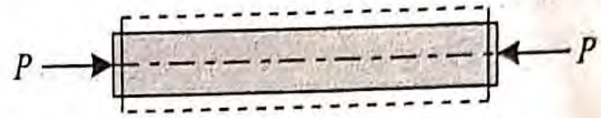


Fig. : Compressive stress

Compressive Stress

When a section is subjected to two equal and opposite pushes and the body tends to shorten its length, as shown in Fig. 28.2, the stress induced is called compressive stress. The corresponding strain is called compressive strain. As a result of the compressive stress, the cross-sectional area of the body gets increased.

Elastic Limit

We have already discussed that whenever some external system of forces acts on a body, it undergoes some deformation. If the external forces, causing deformation, are removed the body springs back to its original position. It has been found that for a given section there is a limiting value of force up to and within which, the deformation entirely disappears on the removal of force. The value of intensity of stress (or simply stress) corresponding to this limiting force is called elastic limit of the material.

Beyond the elastic limit, the material gets into plastic stage and in this stage the deformation does not entirely disappear, on the removal of the force. But as a result of this, there is a residual deformation even after the removal of the force.

Hooke's Law**

It states that, "*When a material is loaded, then within its elastic limit, the stress is proportional to the strain.*"

Mathematically, $\text{Stress} \propto \text{Strain}$

or $$\frac{\text{Stress}}{\text{Strain}} = \text{constant}$$

It may be noted that Hooke's Law equally holds good for tension as well as compression.

Young's Modulus or Modulus of Elasticity (*E*)

We have already discussed that whenever a given material is loaded, then within its elastic limit, the stress is proportional to strain. In other words the ratio of stress and strain for a given material is constant.

This constant is known as Young's Modulus or Modulus of elasticity for a given material and is denoted by *E*. Mathematically,

$$\text{Young's Modulus} = \frac{\text{Stress}}{\text{Strain}}$$

or

$$E = \frac{\sigma}{\varepsilon}$$

where σ = Stress,

ε = Strain and

E = A constant of proportionality known as Young's modulus or modulus of of elasticity. Numerically, it is that value of tensile stress, which when applied to a uniform bar will increase its length to double the original length if the material of the bar could remain perfectly elastic throughout such an excessive strain.

Young's Modulus Appratus

Table

The values of *E* *i.e.*, Young's Modulus or Modulus of Elasticity of some materials used in everyday life, are given below :

S. No.	Material	Young's Modulus of Elasticity (*E*) in units GPa or GN/m^2 or kN/mm^2
1.	Steel	200 to 220
2.	Wrought iron	190 to 200
3.	Cast iron	100 to 160
4.	Copper	90 to 110
5.	Brass	80 to 90
6.	Aluminium	60 to 80
7.	Timber	10

Deformation of a Body Due to Force Acting on it

Consider a body subjected to a tensile stress.

Let P = Load or force acting on the body,

l = Length of the body,

A = Cross-sectional area of the body,

σ = Stress induced in the body,

E = Modulus of elasticity for the material of the body,

ε = Strain, and

δl = Deformation of the body.

We know that the stress

$$\sigma = \frac{P}{A} \qquad \text{Strain,} \qquad \varepsilon = \frac{\sigma}{E} = \frac{P}{AE}$$

and deformation,

$$\delta l = \varepsilon . l = \frac{\sigma . l}{E} = \frac{Pl}{AE} \qquad \ldots \because \sigma = \frac{P}{A}$$

Notes: 1. The above formula holds good for compressive stress also.

2. For most of the structural materials, the modulus of elasticity for compression is the same as that for tension.

3. Sometimes in calculations, the tensile stress and tensile strain are taken as positive, whereas compressive stress and compressive strain as negative.

Example 28.1. *A steel rod 1 m long and 20 mm × 20 mm in cross-section is subjected to a tensile force of 40 kN. Determine the elongation of the rod, if modulus of elasticity for the rod material is 200 GPa.*

Solution. Given : Length (l) = 1 m = 1×10^3 mm ; Cross-sectional area (A) = 20 × 20 = 400 mm^2 ; Tensile force (P) = 40 kN = 40×10^3 N and modulus of elasticity (E) = 200 GPa = 200×10^3 N/mm^2.

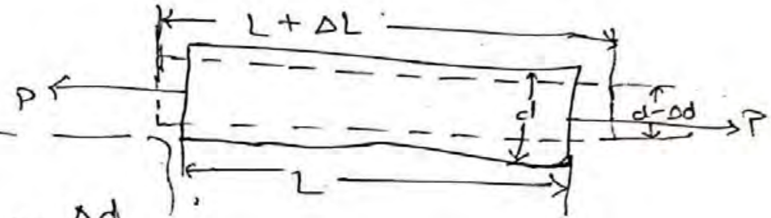
We know that elongation of the rod,

$$\delta l = \frac{P.l}{A.E} = \frac{(40 \times 10^3) \times (1 \times 10^3)}{400 \times (20 \times 10^3)} = 5 \text{ mm} \qquad \textbf{Ans.}$$

Note: In case of taper shaped body, the cross sectional area will be

$$A = (\pi/4) D_1 D_2$$

Poisson's ratio:-



$$\text{lateral strain} = \frac{\Delta d}{d}$$

$$\text{longitudinal strain} = \frac{\Delta L}{L}$$

$$\therefore \text{Poisson's ratio} = \frac{\text{lateral strain}}{\text{longitudinal strain}}$$

$$\therefore \nu = \frac{\Delta d / d}{\Delta L / L}$$

Elastic constants and their Relationship: The various elastic constants are "Modulus of Elasticity (E)", "Sheer Modulus (G)" and "Bulk Modulus (K)". All these elastic constants can be calculated by using the expressions as given:

$$E = \frac{\text{Normal stress}}{\text{" strain}} = \text{modulus of elasticity or Young modulus } (N/mm^2)$$

$$G = \frac{\text{shear stress}}{\text{Shear strain}} = \text{Sheer modulus or Rigidity modulus } (N/mm^2)$$

$$K = \text{Bulk modulus} = \frac{\text{volumetric stress}}{\text{Volumetric strain}} \quad (N/mm^2)$$

$$E = 3K(1-2\nu)$$

$$E = 2G(1+\nu)$$

$$E = \frac{9KG}{G+3K}$$

Q:- A speciman of steel 50mm in dia. & 1000 mm long, was subjected to an axial tantion of 200 KN. Change in length was recorded as 0.509 mm and change in dia 0.007653 mm. Find out value of elastic constant ~~ant~~ E, G, K & ν.

Soln:-

$d = 50 mm$

$L = 1000 mm$

$P = 200 \times 10^3 N$

$\Delta L = 0.509 mm.$

$\Delta d = 0.007653 mm$

Poisson's ration

$$\nu = \frac{\text{lateral strain}}{\text{longitudinal strain}}$$

$$= \frac{\Delta d / d}{\Delta L / L}$$

$$\boxed{\nu = .3}$$ Ans

$$\Delta L = \frac{PL}{AE}$$

$$E = \frac{PL}{\frac{\pi d^2}{4} \times \Delta L} = \frac{200 \times 10^3 \times 1000 \times 4}{\pi \times 50 \times 50 \times .509}$$

$$= 200116.23 \text{ N/mm}^2$$ Ans

$$E = 2G(1+\nu)$$

$$(200116.23) = 2G(1+.3)$$

$$G = 76967.78267 \text{ Ans}$$

$$E = \frac{9KG}{G+3K}$$

$$200116.23 = \frac{9 \times K \times 76967.78267}{76967.78267 + 3K}$$

$$K = 166763 \text{ N/mm2 Ans}$$

Stress- Strain Curve for Ductile Materials

Ductile materials undergo a large amount of deformation before failing. The following figure shows the stress-strain curve for a ductile specimen:

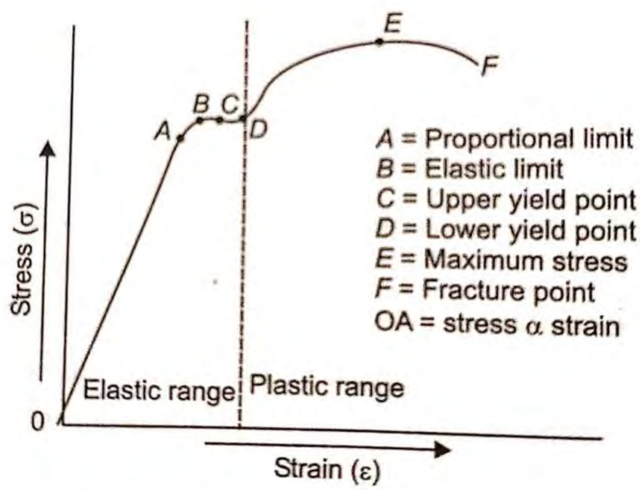


Fig. 10.5: Stress-strain curve.

The points on the graph are explained below:

Proportional Limit: This limit is represented by point A on the graph. Up to this limit, the stress and the strain induced in the specimen are directly proportional to each other, i.e. the specimen obeys Hooke's law. Beyond this point, the stress is not proportional to the strain.

Elastic Limit: This limit is represented by point B on the graph. Upto this limit, the material is said to be elastic. This implies that the specimen regains its original shape and dimensions after the removal of the external load. There are no residual deformations seen in the specimen, on removal of the load. After this point, the material is said to become plastic.

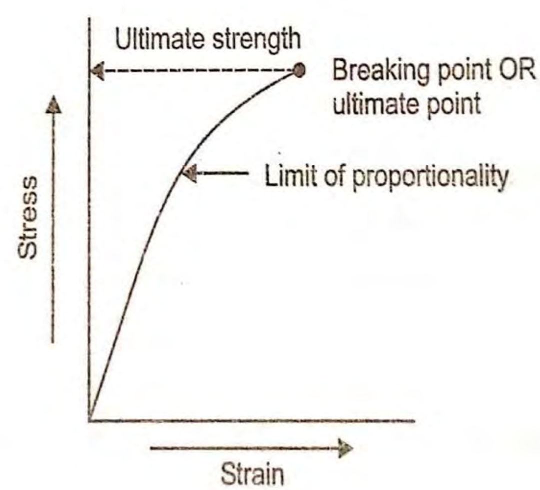
Yield Point: Contrary to what the name suggests, this is a region rather than a point. It is limited by the upper yield point 'C' and the lower yield point 'D'. The stress – strain curve in this part of the graph is almost horizontal, which implies that there is an appreciable increase in strain for a negligible increase in stress. Yielding starts at 'C' and ends at 'D'. After the point 'D', the material, due to strain hardening again starts taking load and the curve rises, as seen in the figure. The material now is said to be plastic and the deformation is of nearly permanent nature.

Ultimate Stress: This is shown by the point 'E' on the graph. It represents the maximum stress that a material can take before it fails. The specimen however does not fail at this point. After this point, the curve starts dropping.

Breaking Point: This is the point at which the specimen fails. After the ultimate stress point, necking of the specimen takes place, which causes a loss in the load carrying capacity of the specimen and ultimately causes it to fail. This point is represented on the curve, by point 'F'.

STRESS - STRAIN CURVE FOR BRITTLE MATERIALS

Materials which show very small elongation before they fracture are called brittle materials. The shape of the curve for a brittle material is shown in Fig. 10.7.



Factor of Safety in Design and Engineering:

Perhaps one of the most important qualities to be considered when creating parts or products is safety...and naturally, an entire industry has cropped up around the need to manufacture safe products and structures for consumer use. Most commonly, you'll hear the terms "Factor of Safety" (FoS) or "Safety Factor (SF), which usually refers to one of two things:

1) the actual load-bearing capacity of a structure or component, or 2) the required margin of safety for a structure or component according to code, law, or design requirements.

A very basic equation to calculate FoS is to divide the ultimate (or maximum) stress by the typical (or working) stress.

A FoS of 1 means that a structure or component will fail exactly when it reaches the design load, and cannot support any additional load.

Structures or components with FoS < 1 are not viable; basically, 1 is the minimum. With the equation above, an FoS of 2 means that a component will fail at twice the design load, and so on.

Q1.- A mild steel speciman of 50 mm diameter and 1000 mm length, was subjected to an axial tension of 200 KN. Find the change in length if modulus of elasticity is 2×10^5 N/mm².

Solⁿ:-

$D = 50mm$

$L = 1000 \, mm$

$P = 200KN = 200 \times 10^3 \, N$

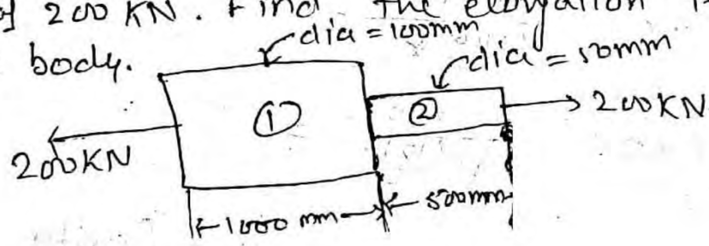
$E = 2 \times 10^5 \, N/mm^2$

$$\Delta L = \frac{PL}{AE} = \frac{4PL}{\pi D^2 E}$$

$$= \frac{4 \times 200 \times 10^3 \times 1000}{\pi \times 50 \times 50 \times 2 \times 10^5}$$

$$= .509 \, mm \quad Ans$$

Q2→ A body shown in fig. is subjected to a load of 200 KN. Find the elongation produced in the body.



$E_1 = 2 \times 10^5 \, N/mm^2$

$E_2 = 1 \times 10^5 \, N/mm^2$

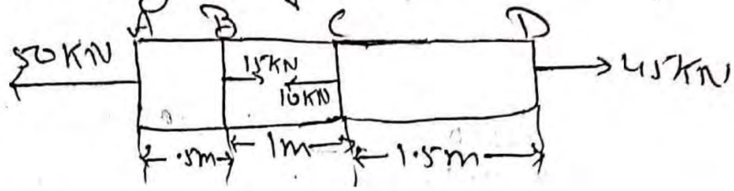
$$\Delta L = \Delta L_1 + \Delta L_2 \quad \text{—— (1)}$$

$$\Delta L = \frac{4 P_1 L_1}{\pi D_1^2 E_1} + \frac{4 \times P_2 L_2}{\pi D_2^2 E_2}$$

$$= \frac{4 \times 200 \times 10^3 \times 1000}{\pi \times (100)^2 \times 2 \times 10^5} + \frac{4 \times 200 \times 10^3 \times 500}{\pi \times (50)^2 \times 1 \times 10^5}$$

$$= 0.636 \, mm \quad Ans$$

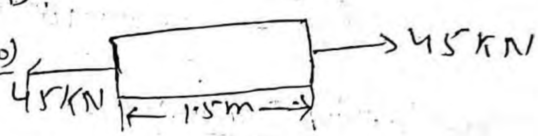
Q:→ A bar of cross section 500 mm² is acted upon by forces as shown. Determine total elongation of bar.



$E = 200 \text{ GPa}$

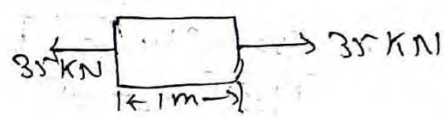
considering Part CD.

$$\Delta L_1 = \frac{(45 \times 10^3)(1.5 \times 1000)}{(500)(2 \times 10^5)}$$

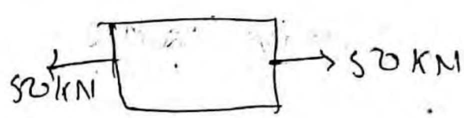


Part BC

$$\Delta L_2 = \frac{(35 \times 10^3)(1 \times 1000)}{(500)(2 \times 10^5)}$$



Part AB



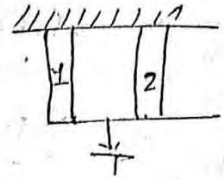
$$\Delta L_3 = \frac{(50 \times 10^3)(.5 \times 1000)}{500 \times 2 \times 10^5}$$

$$\text{Total } \Delta L = \Delta L_1 + \Delta L_2 + \Delta L_3$$

$$= 1.275 \text{ mm.}$$

Problems on Composite bars :-
↳ The bars which is made of combination of more than One metarial.

Eg :-



$$P_1 + P_2 = P \quad \text{---} (1)$$

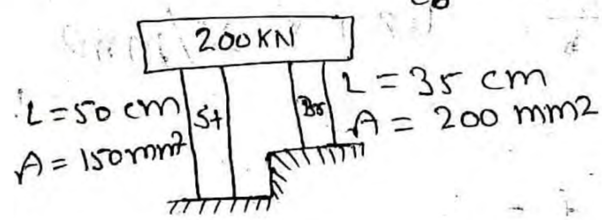
$$\Delta L_1 = \Delta L_2$$

$$\frac{P_1 L_1}{A_1 E_1} = \frac{P_2 L_2}{A_2 E_2} \quad \text{---} (2)$$

$$\sigma_1 = \frac{P_1}{A_1}$$

$$\sigma_2 = \frac{P_2}{A_2}$$

Q :→ A rigid bar is supported by two rods as shown in fig. find the stresses developed in each road, if $\frac{E_s}{E_b} = 2$



Soln :- Let Load shared by steel rod = P_s
Let " " " Brass " = P_b

Hence $P_s + P_b = 200 \times 10^3$ ——— (1)

&

$$\Delta L_s = \Delta L_b$$

$$\frac{P_s L_s}{A_s E_s} = \frac{P_b L_b}{A_b E_b}$$

$$\frac{P_s \times 500}{150 \times E_s} = \frac{P_b \times 350}{200 \times E_b}$$

$$\frac{P_s}{P_b} = \frac{350 \times 3}{200 \times 10} \cdot \frac{E_s}{E_b}$$

$$\frac{P_s}{P_b} = \frac{350 \times 3 \times 2}{200 \times 10}$$

$$P_s = 1.05\, P_b \quad \text{—— (2)}$$

from (1) & (2)

$$\boxed{\begin{aligned} P_s &= 102.5 \text{ KN} \\ P_b &= 97.5 \text{ KN.} \end{aligned}}$$

stress in steel rod

$$\sigma_s = \frac{P_s}{A_s} = 680 \text{ N/mm}^2$$

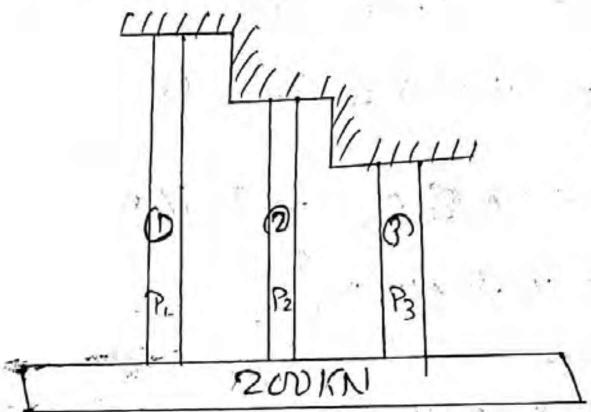
Stress in Brass

$$\sigma_b = \frac{P_b}{A_b} = 487.5 \text{ N/mm}^2$$

√8

Q:- find stresses in each of roads as shown in fig. Given that

	Rod (1)	Rod (2)	Rod (3)
Length (mm)	200	150	100
Area of cross section (mm^2)	500	700	900
E (N/mm^2)	1×10^5	2×10^5	3×10^5



Soln Let load shared by part (1), (2) & (3) are P_1, P_2, & P_3 respectively

$$P_1 + P_2 + P_3 = 200\times10^3 \quad \text{---} (1)$$

$$\Delta L_1 = \Delta L_2 = \Delta L_3$$

i) $\Delta L_1 = \Delta L_2$

$$\frac{P_1 L_1}{A_1 E_1} = \frac{P_2 L_2}{A_2 E_2}$$

$$\frac{P_1 \times 200}{500\times1\times10^5} = \frac{P_2\times150}{700\times2\times10^5}$$

$$\frac{P_1}{P_2} = \frac{\cancel{150}^{75}\times\cancel{500}\times\cancel{10^5}}{\cancel{700}\times\cancel{2}\times\cancel{10^5}\times\cancel{200}} = \frac{15}{56}$$

$$P_1 = .2678\, P_2$$

$$P_2 = 3.734\, P_1$$

$$\Delta L_2 = \Delta L_3$$

$$\frac{P_2 \times 150}{700 \times 2 \times 10^5} = \frac{P_3 \times 100}{900 \times 3 \times 10^5}$$

$$\frac{P_2}{P_3} = \frac{\cancel{100} \times 700 \times 2 \times \cancel{10^5}}{150 \times \cancel{900} \times 3 \times \cancel{10^5}}$$

$$P_2 = .34567\, P_3$$

$$3.734\, P_1 = .34567\, P_3$$

$$P_3 = 10.8\, P_1 \quad \text{--- } ③$$

from eqn ①

$$P_1 + 3.7\, P_1 + 10.8\, P_1 = 200 \times 10^3$$

$$P_1 = 12.8 \times 10^3 N$$

$$P_2 = 3.7 \times 12.8 \times 10^3 N$$

$$P_3 = 10.8 \times 12.8 \times 10^3 N$$

Stress in rod ①

$$\sigma_1 = \frac{P_1}{A_1} = \frac{12.8 \times 10^3}{500}$$

$$= 25.6\ N/mm^2$$

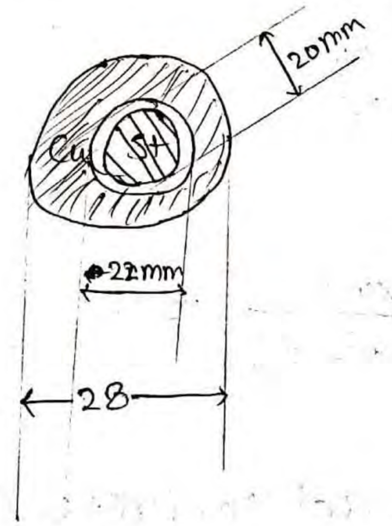
$$\sigma_2 = 68.6\ N/mm^2$$

$$\sigma_3 = 183.6\ N/mm^2$$

Q1- A mild steel Rod 20 mm Diameter passes centerally through a copper tube whose internal dia is 22 mm. & thickness 3 mm. The composite section is 600mm long, & their ends are rigidly connected. If a load of 50 KN acts on the composite section, determine the stresses, in the two meterials and their elongation. Take $E_s = 205\ GN/m^2 = 2.05 \times 10^5\ N/mm^2$

& $E_c = 102.5\ GN/m^2 = 1.025 \times 10^5\ N/mm^2$

Soln



$$P_c + P_s = 50 \times 10^3\ N \quad \text{——} (1)$$

$$\delta L_c = \delta L_s$$

$$\frac{P_c \cancel{L_c}}{A_c E_c} = \frac{P_s \cancel{L_s}}{A_s E_s} \qquad [\because L_s = L_c]$$

$$P_c = P_s \left(\frac{A_c}{A_s}\right)\left(\frac{E_c}{E_s}\right)$$

$$= P_s \left[\frac{\frac{\pi}{4}\{(28)^2 - (22)^2\}}{\frac{\pi}{4}(20)^2}\right]\left[\frac{1.025 \times 10^5}{2.05 \times 10^5}\right]$$

$$P_c = .38\ P_s \quad \text{——} (2)$$

From eqn (1)

$$38P_s + P_s = 50 \times 10^3$$

$$\boxed{P_s = 36.23 \times 10^3 \text{ N}}$$

$$\boxed{P_c = 13.6 \times 10^3 \text{N}}$$

$$\sigma_s = \frac{P_s}{A_s}$$

$$= \frac{36.3 \times 10^3}{\frac{\pi}{4}(20)^2} =$$

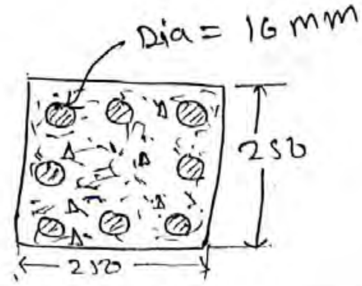
$$\sigma_c = \frac{P_c}{A_c}$$

$$= \frac{13.6 \times 10^3}{\frac{\pi}{4}(28^2 - 22^2)} =$$

Q.~~Imp~~ A rein forced concrete caloum 250 mm × 250 mm in cross section is rein forced with 8 steel bars of 16 mm dia. The caloumn carries an amial load of 270 KN. If modular ratio $\frac{E_{st}}{E_{conc}} = 18$, find stresses in concrete & steel.

If the stress in concrete shall not emceed $4 N/mm^2$, find dia of steel bars, Required so that the caloumn maybe support a load of 400 KN.

Soln :-



Load carried by steel = P_s

" " " concrete = P_c

$$P_s + P_c = 270 \times 10^3 \quad \text{—— (1)}$$

$$\Delta L_s = \Delta L_c$$

$$\frac{P_s \cancel{L_s}}{A_s E_s} = \frac{P_c \cancel{L_c}}{A_c E_c} \qquad [L_s = L_c]$$

$$\frac{P_s}{P_c} = \frac{A_s E_s}{A_c E_c}$$

$$= \frac{8 \times \pi \frac{(16)^2}{4}}{(250 \times 250 - A_s)} \times 18$$

$$= \frac{1608.50}{60891.50} \times 18$$

$$\frac{P_s}{P_c} = .48$$

$$P_s = .48 P_c \quad \text{—— (2)}$$

From (1)

$$.48 P_c + P_c = 270 \times 10^3$$

$$P_c = 182.43 \times 10^3 \text{ N.}$$

$$P_s = 87.57 \times 10^3 \text{ N}$$

$$\sigma_s = \frac{P_s}{A_s} = 50 \text{ N/mm}^2$$

$$\sigma_c = 3.006 \text{ N/mm}^2 = \frac{P_c}{A_c}$$

If load is 400 KN

$$P_s + P_c = 400 \times 10^3 \quad \text{—— (i)}$$

Let dia of steel bar = d

Given $\sigma_c = 4\ N/mm^2$

$$\frac{P_c}{A_c} = 4$$

where $A_c = (250)^2 - \frac{\pi d^2}{4} \times 8$

$$\Rightarrow P_c = 4 A_c$$

$$= 4\left[(250)^2 - 2\pi d^2\right] \quad \text{—— (2)}$$

$$\Delta L_c = \Delta L_s$$

$$\frac{P_c L_c}{A_c E_c} = \frac{P_s L_s}{A_s E_s}$$

$$\frac{P_c}{P_s} = \frac{A_c}{A_s}\left(\frac{E_c}{E_s}\right)$$

$$\left(\frac{P_c}{A_c}\right)\left(\frac{E_s}{E_c}\right) = \frac{P_s}{A_s}$$

$$4 \times 18 = \frac{P_s}{A_s}$$

$$P_s = 72 A_s \quad \text{—— (3)}$$

from (1)

$$72 A_s + 4\left[(250)^2 - 2\pi d^2\right] = 400 \times 10^3$$

$$72 \times \frac{8 \times \pi d^2}{4} + 4\left[(250)^2 - 2\pi d^2\right] = 400 \times 10^3$$

$$144\pi d^2 + 4(250)^2 - 8\pi d^2 = 400 \times 10^3$$

$$136\pi d^2 = 150000$$

$$\boxed{d = 18.74\,mm}$$